LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2024**

UST 3502 – MATRIX AND LINEAR ALGEBRA

Date: 17-04-2024 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Definitions**
a)	Trace of a matrix.
b)	Fundamental system of solutions.
c)	Linear independent set of vectors.
d)	Matrix polynomials.
e)	Congruence of matrices.
2.	**Fill in the blanks**
a)	The transpose of a cofactor matrix of the square matrix is called the ___________.
b)	A system of linear equations $AX = B$ is said to be inconsistent if the system has _________.
c)	Any ordered n-tuples of numbers is called an __________.
d)	If λ is an eigenvalue of a non-singular matrix A then an eigenvalue of A^{-1} is _____.
e)	The ranges of values of two congruent quadratic forms are the ___________.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**True or False**
a)	The diagonal elements of a skew-symmetric matrix are all non-zero.
b)	'A' is an $m \times n$ matrix then the determinant of every square matrix of 'A' is called a major of matrix 'A'.
c)	The rank of the sum of two matrices cannot exceed the sum of their ranks.
d)	The characteristic roots of the skew-Hermitian matrix are either pure imaginary or zero.
e)	Every matrix congruent to a symmetric matrix is a symmetric matrix.
4.	**Answer the following**
a)	If A is a Hermitian matrix, show that iA is skew Hermitian.
b)	Find the rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$
c)	Show that the vectors $X_1 = (1,2,3), X_2 = (2,-2,0)$ form a linearly independent set.
d)	Determine the eigenvalues of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.
e)	Write the matrix form for the following quadratic form : $x_1^2 - 18x_1x_2 + 5x_2^2$
	SECTION B - K3 (CO2)
	Answer any TWO of the following **(2 x 10 = 20)**
5.	(i) If $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, find AA' and $A'A$ (ii) Show that $\Delta \equiv \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix} = (x - 2y + z)^2$

6.	Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ and verify $A\,(adj\ A) = (adj\ A)A = \lvert A \rvert\, I_n$.
7.	Apply the test of rank to examine if the following equations are consistent: $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$ and if consistent, find the complete solution
8.	Prove that the characteristic roots of a Hermitian matrix are real.

SECTION C – K4 (CO3)

Answer any TWO of the following **(2 x 10 = 20)**

9.	(i) Prove if p and q are two scalars and A is any $m \times n$ matrix, then $((p+q)A = pA + qA$. (ii) Prove if A and B are two matrices each of the type $m \times n$, then $k(A+B) = kA + KB$.
10.	Find the rank of the matrix $A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$
11.	Solve completely the system of equations: $x + y + z = 0$, $2x - y - 3z = 0$, $3x - 5y + 4z = 0$, $x + 17y + 4z = 0$
12.	Explain the linear transformation of quadratic form.

SECTION D – K5 (CO4)

Answer any ONE of the following **(1 x 20 = 20)**

13.	(i) Prove that $\Delta \equiv \begin{vmatrix} a^3 & 3a^2 & 3a & 1 \\ a^2 & a^2+2a & 2a+1 & 1 \\ a & 2a+1 & a+2 & 1 \\ 1 & 3 & 3 & 1 \end{vmatrix} = (a-1)^6$ (ii) Solve the following system of linear equation with the help of Cramer's rule $x + 2y + 3z = 6$, $2x + 4y + z = 7$, $3x + 2y + 9z = 14$.
14.	(i) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ (ii) Determine a non-singular matrix Psuch that $P'AP$ is a diagonal matrix, where $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$

SECTION E – K6 (CO5)

Answer any ONE of the following **(1 x 20 = 20)**

15.	(i) Prove that $\Delta \equiv \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = (y-z)(z-x)(x-y)(yz+zx+xy)$. (ii) Find the inverse of the matrix $S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and show that SAS^{-1} is a diagonal matrix where $A = \frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$.
16.	(i) Obtain the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and verify Cayley- Hamilton theorem and find the inverse of the matrix A.

(ii) Reduce the following quadratic form to canonical form and find its rank and signature:

$$x^2 + 4y^2 + 9z^2 + t^2 - 12yz + 6zx - 4xy - 2xt - 6zt.$$